Mathematical Proof (Higher)

THIRD SPACE LEARNING

Odd and even numbers

a) Prove that the sum of two even numbers is even.

If $n \neq m$, $2n$ and $2m$ are distinct even numbers.

$2n + 2m = 2(m + n)$ which is a multiple of 2, so is always even

b) Prove that the product of two distinct odd numbers is also odd.

If $n \neq m$, $2n - 1$ and $2m - 1$ are distinct.

$(2n - 1)(2m - 1)$

$= 4mn - 2m - 2n + 1$

$= 2(2mn - m - n) + 1$

Since $2(2mn - m - n)$ is even, $2(2mn - m - n) + 1$ must be odd

c) Prove that the square of an odd number is also odd.

$(2n - 1)^2$

$= 4n^2 - 4n + 1$

$= 4(n^2 - n) + 1$

Since $4(n^2 - n)$ is even, $4(n^2 - n) + 1$ must be odd

d) Prove that the product of an even number and an odd number is even.

Let $2n$ be an even number and $2m - 1$ be an odd number.

$2n(2m - 1)$

$= 4mn - 2n$

$= 2(2mn - n)$ which is a multiple of 2, so is always even

e) Prove that the sum of two odd numbers is even.

$(2n - 1) + (2m - 1)$

$= 2n + 2m - 2$

$= 2(n + m - 1)$ which is a multiple of 2, so is always even

f) Prove that the sum of an even number and an odd number is odd.

Let $2n$ be an even number and $2m - 1$ be an odd number.

$2n + (2m - 1)$

$= 2n + 2m - 1$

$= 2(n + m) - 1$

Since $2(n + m)$ is even, $2(n + m) - 1$ must be odd

Consecutive numbers

a) Given that n, $n + 1$ and $n + 2$ are consecutive numbers. How would you write three consecutive even numbers?

$2n,\ 2n + 2,\ 2n + 4$

b) Prove that the sum of three consecutive odd numbers is odd.

Let $2n + 1,\ 2n + 3,\ 2n + 5$ any three consecutive odd numbers,

$(2n + 1) + (2n + 3) + (2n + 5)$

$= 6n + 9$

$= 6n + 8 + 1$

$= 2(3n + 4) + 1$

Since $2(3n + 4)$ is even, $2(3n + 4) + 1$ must be odd

c) Prove that the sum of four consecutive even numbers is divisible by four.

Let $2n,\ 2n + 2,\ 2n + 4,\ 2n + 6$ be four consecutive even numbers,

$(2n) + (2n + 2) + (2n + 4) + (2n + 6)$

$= 8n + 12$

$= 4(2n + 3)$ which is always divisible by 4

Direct proof of a statement

Prove that $(n + 3)^2 - (n + 1)^2$ is a multiple of four.

$(n + 3)^2 - (n + 1)^2$

$= n^2 + 6n + 9 - n^2 - 2n - 1$

$= 4n + 8$

$= 4(n + 2)$ which is a multiple of four

Using algebra skills

Prove that $2x^2 - 6x + 5$ is always positive.

Complete the square,

$2x^2 - 6x + 5 = 2(x - \frac{3}{2})^2 + \frac{1}{2}$ and since $(x - \frac{3}{2})^2$ is always positive, $2(x - \frac{3}{2})^2 + \frac{1}{2}$ is always positive.

Giving a counterexample

Haz thinks that every term generated by $u_n = 4n - 1$ is a prime number. Find a counterexample to show that Haz is incorrect.

$u_4 = 4(4) - 1 = 15$, which is not a prime number

(Other counterexamples exist)